Practical Considerations in Optimal Flight Management Computations

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The relationships between cost index, speed, flight time, fuel burn, and wind are investigated via simulations of flight of a DC-10 aircraft, as controlled by a flight management computer system, over a representative 1000-n.mi. flight plan. In this context, a two-part procedure is described for achieving minimum total flight cost, that is, direct operating cost plus arrival error cost. First the optimum cost index and the associated optimum arrival time (which sometimes turns out to be the scheduled arrival time) are sought. A simple method for fine-tuning the speed to achieve high arrival time accuracy is then described. The methods are practical to incorporate in flight management computer systems.

Introduction

A PRINCIPAL objective of the flight management computer system (FMCS) is to minimize the cost of flight. The current equipment attempts to achieve this objective by selecting vertical and lateral profiles that minimize direct operating cost (DOC), which is the cost of fuel plus any other cost that is proportional to flight time. A significant share of the savings achieved by the current FMCS is due to effective lateral planning and navigation that result in reduced air distance. The remaining share of the savings is due to optimal selection of cruise altitudes, step climb points, thrust settings, and speed commands.

Flight-time cost, such as crew cost and maintenance and repair cost that may reasonably be prorated with flight time, is represented in the FMCS calculations in terms of a cost index (CI), which is defined as the ratio of time cost to fuel cost. In systems with English units of measure, the CI normally has units of 100 lb/h; in metric systems kg/min is used. (The scaling is chosen to provide three-digit entry on a keyboard.) The CI is a predetermined parameter that theoretically should be fixed for a given aircraft but that may be set by the pilot to any value from 0 to 999.

The FMCS then attempts to compute the climb, cruise, and descent profiles that minimize the direct operating cost, expressed as

$$J_{\mathrm{DOC}} = C_f \int_0^{t_f} (f+K)\,\mathrm{d}t \tag{1}$$

where

C_f = unit cost of fuel ($/lb or $/kg)
f = fuel flow (lb/h or kg/h)
K = cost index scaled to the units of fuel flow
t_f = flight time

The aircraft is also constrained to fly along a fixed lateral path, at air traffic control (ATC)-approved cruise flight levels, and is subject to airframe and ATC-imposed speed limits.

In this paper we assume that the current FMCS designs adequately solve the problem of computing the optimum speed, thrust, altitude, etc., to minimize DOC. The methods are based on the energy-state approach formulated by Erzberger and Lee,[1] with some practical constraints such as constant CAS/Mach profiles for climb and descent.

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A significant drawback with the DOC approach is that it does not properly reflect costs associated with arrival time error, such as crew overtime cost, losses due to missed connections in a hubbing operation, and potential losses due to customer dissatisfaction with the airline. The airlines have used the CI as a means for adjusting the average flight time for a city pair flight, based on normal wind conditions and schedule considerations. In one application (UAL DC-10) the CI is automatically adjusted in-flight under favorable wind conditions to cause the aircraft to slow down toward the zero CI speed; however, it is not increased above a reference CI for the city pair.[2]

When the CI is used in this way, it no longer represents the flight-time cost initially intended but becomes a means for adjusting arrival time. It has been shown[3,4] that this use of the CI provides the minimum fuel solution for the resulting arrival time. Methods for finding the CI that result in a desired arrival time are therefore a basic issue of this paper.

The four-dimensional (4D) guidance problem is similar, but with different emphasis. A major objective in 4D guidance is to increase the efficiency of air traffic control in a congested terminal area environment by controlling the spacing of aircraft so that overall flow rate is maximized. Ground-based ATC computations generate landing orders and arrival times at a metering fix, and the arrival time at the fix needs to be controlled very accurately (± 5 s is desired). A fundamental problem is the control of a mixture of aircraft types, only some of which may be equipped with 4D guidance capability. Although efficient terminal area control is a very significant factor in minimizing flight cost, this paper does not deal with descent speed strategies or constraints that may be required to maximize terminal area flow rate. It is primarily concerned with adjusting the speed, within prescribed constraints, to achieve high arrival time accuracy at minimum cost. In the 4D case, the arrival time is prescribed; in the minimum cost case, it must be computed.

This paper also does not deal with modifying the path, such as path stretching or early descent, for efficient time absorption. Chakravarty[5] has shown that cruise altitude should be reduced for a large negative CI, and he has developed optimal 4D trajectory algorithms. Some savings are available by such algorithms, but they require too much computation to be implementable on the current or near-term FMCS. Chakravarty[6] has also investigated the problem of total flight cost and described a method for selecting the optimum CI under some assumptions on the shape of the arrival error cost function, namely, linear or parabolic. The method presented here makes no such assumptions, is conceptually and computationally simpler, and includes practical methods of computation by an FMCS.

Simulation Study

In order to better understand the relationships between CI, speed, flight time, fuel burn, and winds, a simulation study of a representative flight of a DC-10 aircraft over a 1000-n.mi. flight plan was undertaken. The aircraft was guided vertically by the FMCS laws that currently rule the UAL DC-10 aircraft. The initial and final altitudes were 1000 ft, and a cruise altitude of 35,000 ft was selected without step climbs or descents. The International Standard Atmosphere was assumed, and the initial gross weight was 350,000 lb. Takeoff and final approach phases were not included in the profiles, and a clean configuration was assumed throughout the flight. The 250 knot ATC speed limit was observed below 10,000 ft, and the aircraft was at speed at the initial and final altitudes.

Three wind conditions were applied: no wind (NW), head wind (HW), and tail wind (TW). The HW and TW conditions have 50 knots magnitude at 35,000 ft, decreasing linearly to 0 knots at sea level.

Simulation runs were made under each wind condition with the following values of CI (in 100 lb/h): −100, −75, −50, −25, −10, 0, 10, 25, 50, 75, 100, 150, 200, 300, 400, 600, 800, 1000. The CI value that represents real flight-time cost usually lies in the range of 25 to 75.

A negative CI value (not selectable on the current FMCS) results in lower speed than the minimum-fuel speed and in more fuel consumption than the minimum for unconstrained arrival time. When time absorption is required, the minimum-fuel solution for the required arrival time is obtained with a negative CI. However, the CI should not be less than the negative of the minimum cruise fuel-flow value (i.e., the speed should not go below the maximum endurance speed). Further time delay should be implemented by increased path length (usually a holding pattern) at maximum endurance speed. (These results are described in Ref. 3.) The maximum endurance speed is not investigated in this study.

Figure 1 shows the effects of the CI on the average true airspeed (air distance divided by flight time) at the three wind conditions and points out the significant nonlinearities. The speed for a given CI depends strongly on the wind, and this dependency is very nonlinear. Also, the speed sensitivity to CI is very high at small and negative values, and very low at large values. So the CI is not a convenient paremeter for controlling airspeed.

Figure 2 shows the effects of the CI on flight time, with the analogous sensitivities at large and small CI values. The high sensitivity to wind is evident, as is the large increase in CI required to make up for an unexpected head wind.

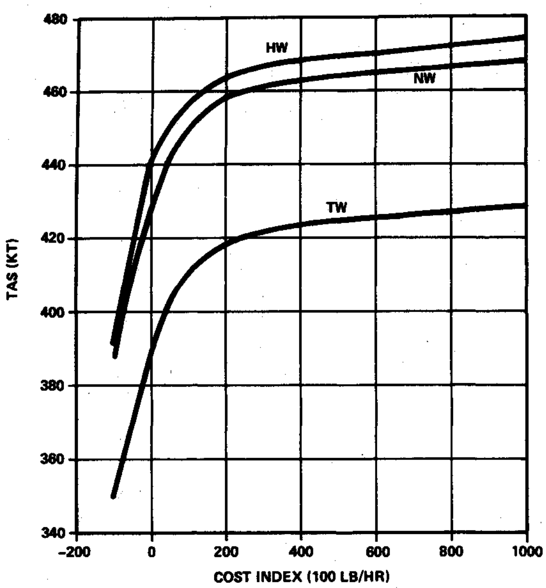

Fig. 1 Average true airspeed vs CI.

Figure 3 shows fuel burn vs CI. Theoretically, CI = 0 should yield minimum fuel burn, but Fig. 3 shows the minimum point to be a slightly negative CI value. A few simplifying assumptions and approximations were made in the FMCS speed command algorithms, however, such as constant CAS/Mach profiles in climb and descent. (Such profiles do not quite yield minimum fuel.)

Figure 4 shows minimum fuel burn vs flight time for the three wind conditions, with the CI points marked. Figure 4 also shows that a flight time of 2 h 20 min results for CI = 0 and no wind. To have the same flight time under a 50-knot head wind requires CI = 900 and approximately 6600 lb of additional of fuel.

It should be pointed out that the results are based on controlling the speed over the entire 1000-n.mi. trip. Also, the cruise altitude and the initial fuel onboard are the same in all cases. Since fuel planning takes wind forecasts into account, the head-wind case would normally have more initial fuel than the tail-wind case, which would tend to separate the curves in

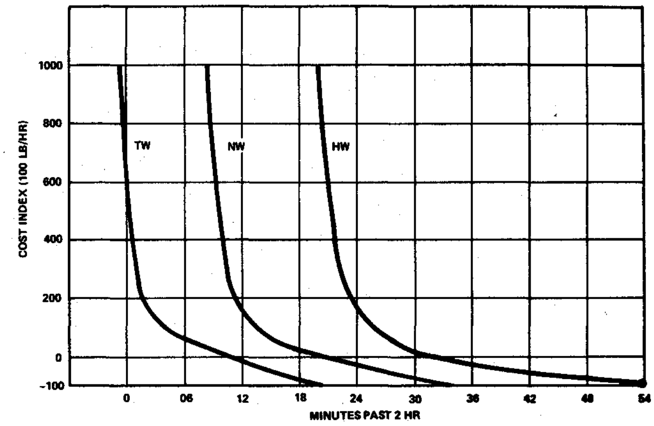

Fig. 2 Flight time vs CI.

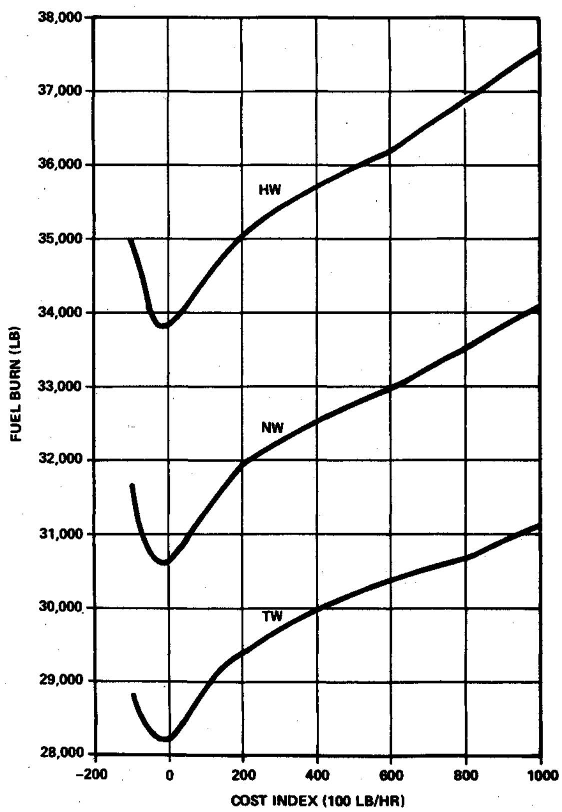

Fig. 3 Fuel burn vs CI.

Fig. 4 even further. A fully optimized trajectory would mitigate the fuel penalty to some degree.

In any case, the curves are obtainable only by doing a prediction (a fast-time simulation of flight from the current aircraft position) to the destination in question for each set of given CI values, taking into account winds, flight plan constraints, guidance laws, etc., at every point along the flight plan. Such predictions consume considerable processing time in an FMCS, since they must accurately simulate aircraft drag, thrust, fuel flow, etc. Furthermore, they must include temporary auxiliary predictions, such as for generating a descent path by backward prediction from the destination way point. Each prediction iteration is costly, and the number of iterations undertaken to search for an optimum CI must be minimized.

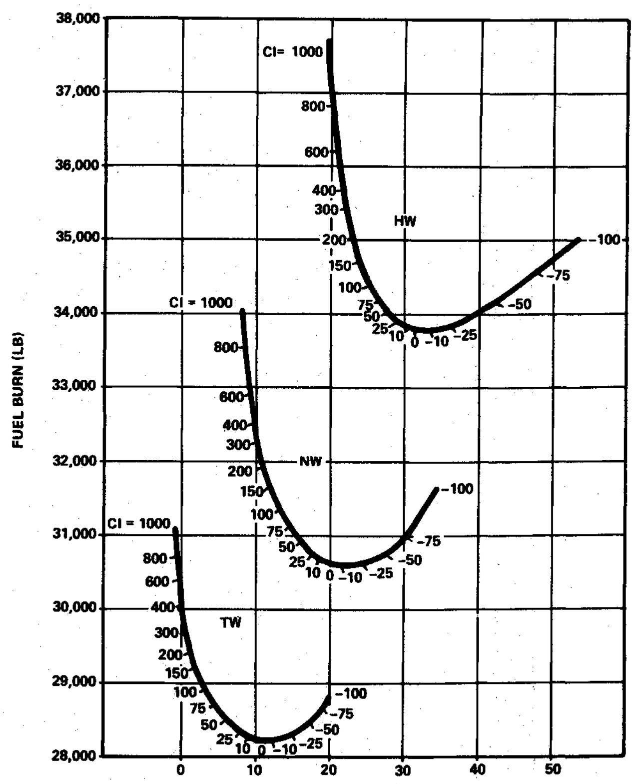

Fig. 4 Minimum fuel vs flight time.

Optimum Cost Index Search

We define total flight cost (TFC) to be the sum of the direct operating cost (DOC) and the arrival error cost (AEC), expressed as

$$J(K)=C_f\int_0^{t_f}[f(K)+K_0]\mathrm{d}t+\phi(t_e) \qquad (2)$$

where

C_f = unit cost of fuel
K = cost index in units of fuel flow (a variable)
K_0 = the value of K that represents the actual flight-hour cost (a constant)
$f(K)$ = the fuel flow consumed at each point in the profile when optimized for cost index K
t_f = the flight time that results for cost index K
t_e = arrival time error (actual arrival time minus scheduled arrival time)
$\phi(t_e)$ = arrival error cost function

As the result of a prediction pass for a given K we have t_f, t_e, and fuel burn F so we can evaluate

$$J(K)=C_f(F+K_0t_f)+\phi(t_e) \qquad (3)$$

Constants C_f and K_0 and function $\phi(t_e)$ must, of course, be provided. The optimum CI is then the one that minimizes J.

The AEC function $\phi(t_e)$ is a composite of several cost elements associated with arrival time error (such as overtime cost, customer dissatisfaction cost, and losses due to missed connections). Overtime cost normally begins at the expiration of the scheduled flight time, which may be later than the scheduled arrival time if the flight starts late. The choice of $\phi(t_e)$ is somewhat arbitrary and is airline dependent. Appropriate functions must be provided but require studies that are beyond the scope of this paper. Figure 5 shows an illustrative example of such a function, made up of a linear component at \$600/h for $t_e>0$, and two steps representing missed connection costs. No cost is assigned to early arrival.

The AEC values are expected to be different for each termination, so a convenient way must be found to store such functions in the FMCS data base and possibly to modify them by keyboard entry. The following format is suggested:

$$\phi(t_e)=\sum_i \sigma(t_i,s_i,m_i) \qquad (4)$$

where $\sigma(t_i,s_i,m_i)$ is a sloping step function of time which steps up for increasing postive t_e, and also for decreasing negative t_e (to allow for cost associated with early arrival). It is defined in terms of step point t_i, step size s_i, and slope m_i by

$$\sigma(t_i,s_i,m_i)=\begin{cases} s_i+m_i(t_i-t_e) & \text{if } t_e<t_i\le 0\\ 0 & \text{if } t_i\le t_e\le 0 \text{ or } 0\le t_e\le t_i\\ s_i+m_i(t_e-t_i) & \text{if } 0\le t_i<t_e\end{cases} \qquad (5)$$

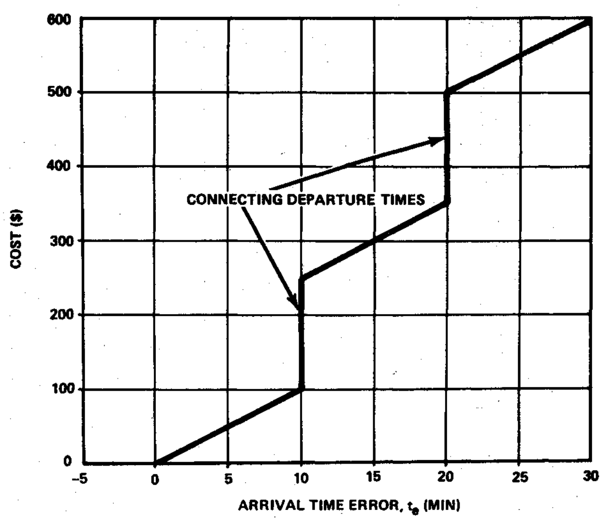

Fig. 5 Arrival error cost function.

The variable t_e is omitted in the σ format for the sake of brevity. The time points $\{t_i\}$ are important in the search for the optimum CI as described below; they are referred to as the AEC critical points. The AEC function in Fig. 5 then can be expressed as

$$\phi(t_e)=\sigma(0,0,10)+\sigma(10,150,0)+\sigma(20,150,0) \qquad (6)$$

A window-type function of width $2t_w$, centered at $t_e=0$ and with cost C_w outside the window, is defined by

$$\sigma(t_e)=\sigma(-t_w,C_w,0)+\sigma(t_w,C_w,0) \qquad (7)$$

Figure 6 shows how total flight cost is constructed. The DOC curve corresponds to the head-wind case described earlier, with $C_f = 13¢/\text{lb}$ and $C_f K_0 = \$650/\text{h}$ (CI = 50). We assume that the scheduled arrival time was based on the no-wind case with CI = 0, which corresponds to a flight time of 2h 20min. The arrival time cost function of Fig. 5 is then referenced to this time, and the total cost is as shown. By inspection we see that the optimum arrival time is around 2:26, or 6 min late, and the optimum CI is in the range 75 to 100. On-time arrival would require a CI of 900 and cost around \$250 more. At CI = 0, a connecting flight is missed and the total cost would be around \$200 more than the optimum.

An important point to note is that sizable variations in CI near the optimum value have a negligible effect on total cost unless the minimum point lies near a critical point. In this example, CI can vary from about 30 to 130 with only a \$10 variation in total cost. This variation in CI corresponds to over 4 min in variation in arrival time (about 3% of the flight time).

For the general case, the procedure for finding the optimum CI is as follows:

1) Find the time t_{opt} that minimizes TFC from a plot of TFC vs flight time (such as Fig. 6).

2) Find the corresponding CI from plot of CI vs flight time (such as Fig. 2).

However, each point on such curves requires a prediction pass for a trial CI. For practical reasons, the number of such predictions must be minimized, so the trial CI values must be carefully chosen. The procedure described below uses CI values from a reference set in which parabolic interpolation is employed to estimate intermediate values. Suggested CI reference values are −100, −50, −25, −10, 25, 150, 400, and 1000.

The following procedure is then used to estimate the optimum CI:

1) Select three CI values from the reference set that are expected to lie in the vicinity of the optimum value. With no advance knowledge, select −10, 25, and 150.

2) Perform predictions with the three CI values to obtain corresponding values for DOC and flight time.

3) Fit a parabola to the three points to approximate DOC vs flight time; then add the AEC function to approximate TFC.

4) Do a simple linear search on TFC vs time to find the minimum point. The search intervals can be fairly large (60 s) but must include the AEC critical points.

5) If the resulting minimum point should lie outside the span of the three prediction times, select a new adjacent trial CI from the reference set, in the appropriate direction, and do another prediction that replaces the previous one farthest removed. Then repeat from step 3.

6) Find the optimum CI by parabolic interpolation on the three prediction values of CI vs flight time.

This procedure represents a practical compromise between accuracy and processing load, based on the observations that TFC is relatively insensitive to CI per se and that sensitivity to arrival time is significant only in the vicinity of AEC critical points. Since such time points are predefined, the accuracy of an optimum point at a critical time is ensured. This procedure will normally require just three prediction passes.

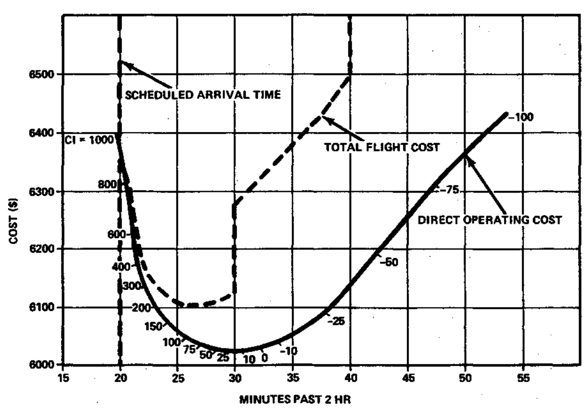

Fig. 6 Flight costs vs flight time.

Fine Speed Adjustment

With an optimal arrival time and a near-optimal cost index, the corresponding speed should result in arrival near the optimal time. However, the approximations are likely to result in inadequate arrival time accuracy. The second part of the procedure is, therefore, to fine-tune the speed obtained with the computed CI in order to achieve high arrival time accuracy. When flight time is fixed, then the average ground speed is fixed. Therefore, the difference between the speed profile with an exact cost index, and one with an inexact cost index plus a small speed adjustment, can only be in the distribution of the speed, and is expected to have negligible cost impact.

The following discussion attempts to explain a speed adjustment algorithm that was developed somewhat heuristically but which has been found to be effective in adjusting the speed to reduce the arrival time error. A method was sought for finding a speed command in the form

$$V_R = V_0 + V_{SA} \tag{8}$$

where V_R is the required true airspeed such that the aircraft arrives at a destination point at a required arrival time T_R and V_0 is the speed command generated by the FMCS, normally the optimum speed for a given CI but often subject to speed and acceleration limits. The V_0 values are computed at every point in the prediction profile by the normal FMCS algorithms. The speed adjustment term V_{SA} should be much simpler to compute. The method is explained as follows.

Let X be the distance to the destination way point along the horizontal flight path. Then

$$X = \int_0^T V_G \mathrm{d}t = \bar{V}_G T \tag{9}$$

where V_G is ground speed, T is flight time to the destination, and $\bar{V}_G$ is the time-averaged V_G. Let $\bar{V}_{G0}$ be the $\bar{V}_G$ for a prediction that uses the FMCS-generated speed command without speed adjustment, let T_0 be the resulting arrival time, let T_R be the required arrival time, and let $\bar{V}_{GR}$ be the $\bar{V}_G$ required to arrive on time. Then

$$X = \bar{V}_{G0} T_0 = \bar{V}_{GR} T_R \tag{10}$$

so

$$\bar{V}_{GR} = (T_0/T_R)\bar{V}_{G0} \tag{11}$$

We now make the assumption that the time average of V_G very nearly equals the distance average; that is,

$$\frac{1}{T}\int_0^T V_G(t)\mathrm{d}t = \frac{1}{X}\int_0^X V_G(x)\mathrm{d}x \tag{12}$$

Then

$$\frac{1}{X}\int_0^X V_{GR}\mathrm{d}x = \frac{1}{X}\int_0^X \frac{T_0}{T_R} V_{G0}\mathrm{d}x \tag{13}$$

which is satisfied by letting

$$V_{GR} = (T_0/T_R)V_{G0} \tag{14}$$

at every distance point along the trajectory. In terms of true airspeed V and tail wind V_w this becomes

$$V_R + V_w = (T_0/T_R)(V_0 + V_w) \tag{15}$$

or

$$V_R = V_0 + (T_0/T_R - 1)(V_0 + V_w) \tag{16}$$

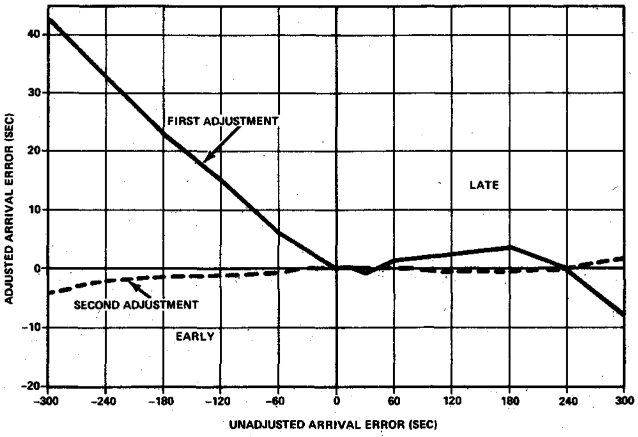

Fig. 7 Speed adjustment results.

By reapplying Eq. (14), we get

$$V_R = V_0 + (\Delta T_0/T_0)V_{GR} \tag{17}$$

Let V_1 denote the first estimate of V_R based on one prediction pass; that is, let

$$V_1 = V_0 + (\Delta T_0/T_0)V_{G1} \tag{18}$$

where T_0 is the arrival time with V_0 as the speed function and $\Delta T_0 = T_0 - T_R$. After another iteration we similarly have

$$\begin{aligned} V_2 &= V_1 + (\Delta T_1/T_1)V_{G2} \\ &= V_0 + (\Delta T_0/T_0)V_{G1} + (\Delta T_1/T_1)V_{G2} \end{aligned} \tag{19}$$

After N predictions, we obtain for the estimate of V_R

$$V_N = V_0 + \sum_{i=0}^{N-1} \frac{\Delta T_i}{T_i} V_{G(i+1)} \tag{20}$$

We now observe that V_G does not change much for small speed corrections, so we replace Eq. (20) by

$$V_N = V_0 + \left(\sum_{i=0}^{N-1} \frac{\Delta T_i}{T_i}\right) V_{GN} \tag{21}$$

Defining the speed adjustment factor as

$$K_{SA} = \sum_{i=0}^{N-1} \frac{\Delta T_i}{T_i} \tag{22}$$

we then have the adjusted speed in the form that was sought:

$$V_R = V_0 + K_{SA} V_{GR} \tag{23}$$

The V_0 value is obtainable at every point from the normal FMCS speed command and K_{SA} is updated at the end of each prediction pass. Here V_{GR} $(= V_R + V_w)$ is the adjusted ground speed at the current prediction position, but the value at the previous interval has been found to be close enough. Alternately, Eq. (23) can be rearranged to give

$$V_R = (V_0 + K_{SA} V_w)/(1 - K_{SA}) \tag{24}$$

where V_w is obtained from a wind forecast model. This form is slightly different from the one initially sought, but it meets the same basic objective of being simple to evaluate at every point.

The formulation so far assumes that the aircraft is able to fly at the adjusted speed. However, speed and acceleration limits prevent speed adjustments over certain segments of the flight, which must be made up for by greater speed adjustments over the remaining segments. We take this into account by including in the prediction algorithm two additional time variables: T_u, the accumulated time during which speed may be increased, and T_d, the accumulated time during which speed may be decreased. The time segments excluded are those where a speed limit, imposed either by the flight plan or by aircraft limits, prevents the speed from being increased or decreased, respectively. Also excluded are the periods where the aircraft is accelerating or decelerating at a limited value. If at the end of the prediction pass the arrival time is found to be late, T_{ui} is used instead of T_i in updating K_{SA} by Eq. (22). Similarly, if the arrival time is found to be early, T_{di} is used instead of T_i.

Figure 7 illustrates some results obtained with this method. The previously described simulation was run with no wind and CI = 0, which results in a flight time of 2 h 19 min 58 s. Fourteen cases of time constraints were run, corresponding to a required arrival time that deviates from the preceding time by ±15 s, ±30 s, ±1 min, ±2 min, ±3 min, ±4 min, and ±5 min. Figure 7 shows the result of the speed adjustment after one and two adjustments. For example, if the nonadjusted speed results in a 3-min-late arrival, then the first adjustment reduced the arrival error to 3.6 s and the second to −0.5 s. The early arrival cases tended to overadjust somewhat on the first adjustment. For example, when the nonadjusted speed corresponded to a 5-min-early arrival the result of the first adjustment was a 43-s-late arrival. The second adjustment resulted in a 4.3-s-early arrival.

These results illustrate how rapidly the speed adjustment algorithm converges. Another important advantage is that it uses the predictions that are already required by the FMCS in normal operation. Hence this method may not require any additional prediction passes. By comparison, the trial predictions to search for the optimum CI are not usable for normal FMCS predictions.

The accuracy in controlling arrival time is, for any algorithm, very dependent on having accurate wind forecast information as well as accurate navigation information. The effect of winds has been studied by Chakravarty[5] and Menga and Erzberger.[7] Future systems, with direct ground-to-air data links (ACARS or Mode S), are likely to provide significant improvements in wind forecasting.

Conclusions

1) Total flight cost, comprised of direct operating cost plus arrival error cost, is relatively insensitive to cost index in the vicinity of the optimum arrival time unless this time is near a critical point in the arrival error cost function (such as the scheduled arrival time or connecting flight times).

2) The optimum arrival time and cost index can be found with adequate accuracy for minimizing cost by a procedure that normally requires just three trial prediction passes.

3) Arrival time can be controlled very accurately by a simple speed adjustment algorithm that requires very few, if any, extra prediction passes.

4) Speed control that minimizes total flight cost or results in accurate arrival time is achievable with relatively modest modification to the current FMCS design. However, valid arrival error cost functions, which are generally different for each arrival situation, must be defined and stored in the FMCS data base.

References

[1]Erzberger, H. and Lee, H., "Constrained Optimum Trajectories with Specified Range," *Journal of Guidance and Control*, Vol. 3, Jan.-Feb. 1980, pp. 78-85.

[2]Velle, J. I. and Walters, C. G., "A Fuel-Efficient Four-Dimensional Guidance System," *Aviation Fuel Conservation Symposium*, Washington D. C., Sept. 1984.

[3]Sorensen, J. A. and Waters, M. H., "Airborne Method to Minimize Fuel with Fixed Time-of-Arrival Constraints," *Journal of Guidance and Control*, Vol. 4, May-June 1981, pp. 348-349.

[4]Burrows, J. W., "Fuel-Optimal Aircraft Trajectories with Fixed Arrival Times," *Journal of Guidance and Control*, Vol. 6, Jan.-Feb. 1983, pp. 14-19.

[5]Chakravarty, A., "4-D Fuel-Optimal Guidance in the Presence of Winds," *AIAA Guidance and Control Conference*, Gatlinburg, TN, Aug. 1983.

[6]Chakravarty, A., "Selection of an Optimal Cost Index for Airline Hub Operation," *Journal of Guidance, Control, and Dynamics*, Vol. 8, Nov.-Dec. 1985.

[7]Menga, G. and Erzberger, H., "Time-Controlled Descent Guidance in Uncertain Winds," *Journal of Guidance and Control*, Vol. 1, March-April 1978, pp. 123-129.